

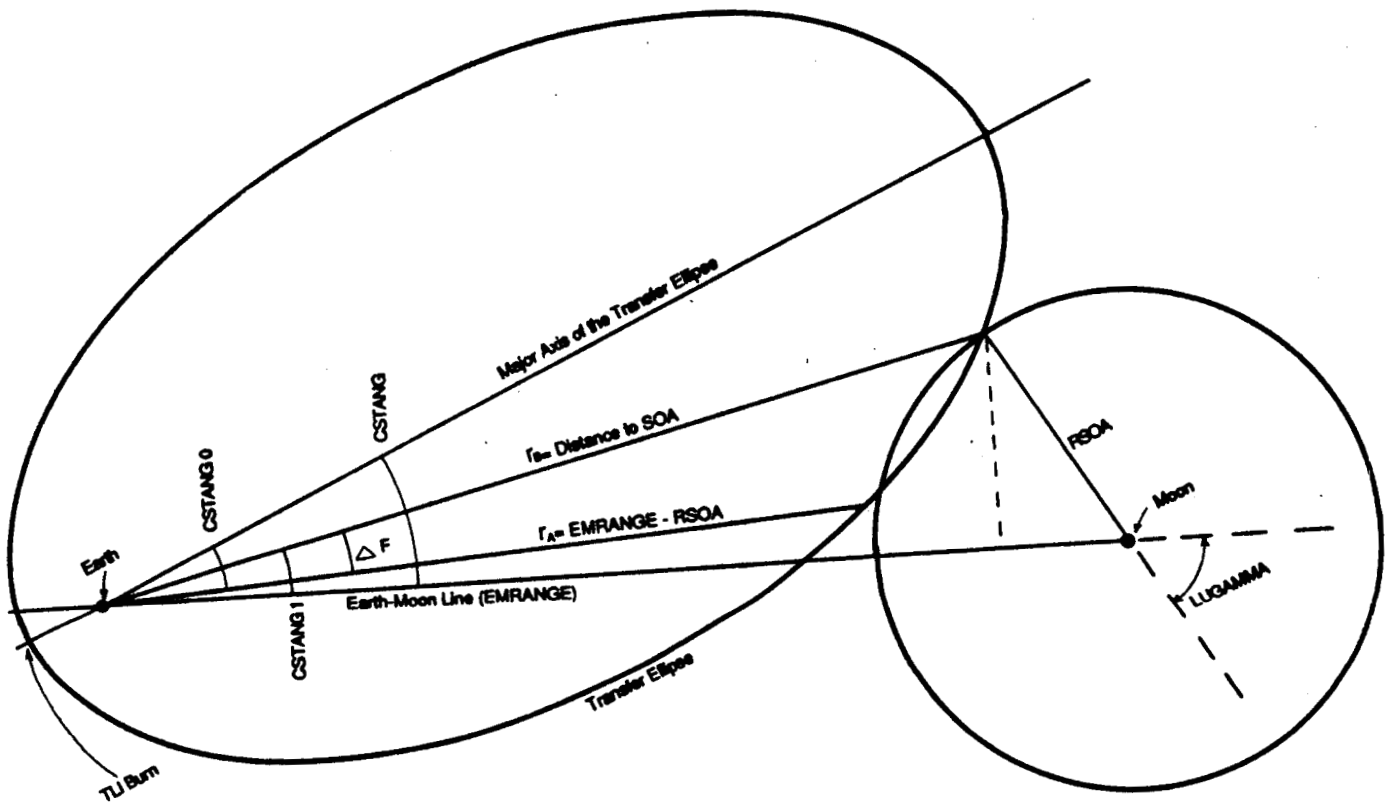


LLOFX

Earth Orbit to Lunar Orbit

Delta V Estimation Program

User and Technical Documentation



(NASA-CR-172091) LLOFX EARTH ORBIT TO LUNAR
ORBIT DELTA V ESTIMATION PROGRAM USER AND
TECHNICAL DOCUMENTATION (Eagle Engineering)
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1.0 Introduction

The program LLOFX calculates in-plane trajectories from an Earth-orbiting space station to Lunar orbit in such a way that the journey requires only two delta-v burns (one to leave Earth circular orbit and one to circularize into Lunar orbit). The program requires the user to supply the space station altitude and Lunar orbit altitude (in kilometers above the surface), and the desired time of flight for the transfer (in hours). It then determines and displays the trans-Lunar injection (TLI) delta-v required to achieve the transfer, the Lunar orbit insertion (LOI) delta-v required to circularize the orbit around the Moon, the actual time of flight, and whether the transfer orbit is elliptical or hyperbolic. Return information is also displayed. Finally, a plot of the transfer orbit is displayed.

2.0 Principle Behind the Program

Calculation of the trajectory takes advantage of the fact that the Moon travels at great velocity in orbit about the Earth (1.02 kilometers per second). The vehicle's circular orbit about the Earth is turned into an elliptical transfer orbit that intercepts the Moon's orbit. This transfer orbit is rotated ahead of the Earth-Moon line in such a way that, as the vehicle enters the Moon's sphere of action (SOA) ahead of the Moon, the high velocity of the Moon in the direction of the vehicle causes the vehicle to appear to be headed back toward the Moon (from a Lunar point of view). This program identifies the eccentricity, size, and rotation of the transfer ellipse or hyperbola that causes the velocity vector of the vehicle (in Lunar coordinates) to correspond to an orbit passing in front of the Moon with a perigee at the Lunar orbit altitude supplied by the user.

3.0 Description of the Process

Given the altitude of the space station circular orbit, the program calculates circular velocity. Through a process of iteration, velocity is added to this in small increments (delta-v) so that the orbit becomes elliptical or hyperbolic. Assuming that the burn

occurs at the far side of the Earth on the Earth-Moon line, such an ellipse/hyperbola will be symmetrical along the Earth-Moon line, as defined at the time of SOA penetration (see Figure 1). The velocity is increased until this trajectory orbit's apogee is beyond the Moon's SOA. (A hyperbolic orbit meets this condition by definition).

Initially, the program identifies the vehicle's velocity vector (in Lunar coordinates) at a point on the transfer orbit such that the distance from the Earth to that point equals the distance from the Earth to the Moon's SOA, measured along the Earth-Moon line. Note that because of the Moon's motion around the Earth, and because we are using Lunar coordinates, the velocity vector points away from the transfer orbit.

The position on the SOA is determined at which the velocity vector identified above would correspond to an orbit passing in front of the Moon with a perigee equal to the user supplied lunar orbit altitude (see Figure 2). The position is identified by an angle centered at the Moon (LUGAMMA), measured up from the Earth-Moon line. The ellipse/hyperbola is rotated through an angle called the coast angle (CSTANG) such that it intersects the SOA at this position. (The vehicle physically performs this by causing the TLI burn to occur at an angle past the Earth-Moon line equal to the coast angle. See Appendix D for a description of how the coast angle is calculated.)

This point of intersection of the SOA and the transfer orbit occurs further along the transfer orbit than the point at which the initial velocity vector was identified. The velocity vector at this new point is different from the originally calculated velocity vector (see Figure 3). This new velocity vector (B in the diagram) is not pointing in a direction that will allow interception with Lunar orbit perigee. A new position on the SOA must be determined to allow this condition to be met (B' in the diagram). Figure 4 shows the calculations required to determine this new position (LUGAMMANEW) given

the velocity vector's x- and y-components and the flight path angle (ANGMOMGAM). (See Appendix E for a description of how ANGMOMGAM is calculated).

The steps described in the previous paragraph must be iterated until LUGAMMANEW converges (to within 0.02 radians). The resulting coast angle describes the point in Earth orbit at which to perform the TLI burn.

The program then adds the time of flight between SOA and Lunar orbit to the time of flight between TLI and SOA to get total time of flight. If the total flight time does not fall within one hour of the desired time of flight, TLI delta-v is adjusted by 1/10th the percentage of error between actual and desired time, and the entire process is begun anew. Otherwise, the program displays the transfer orbit properties.

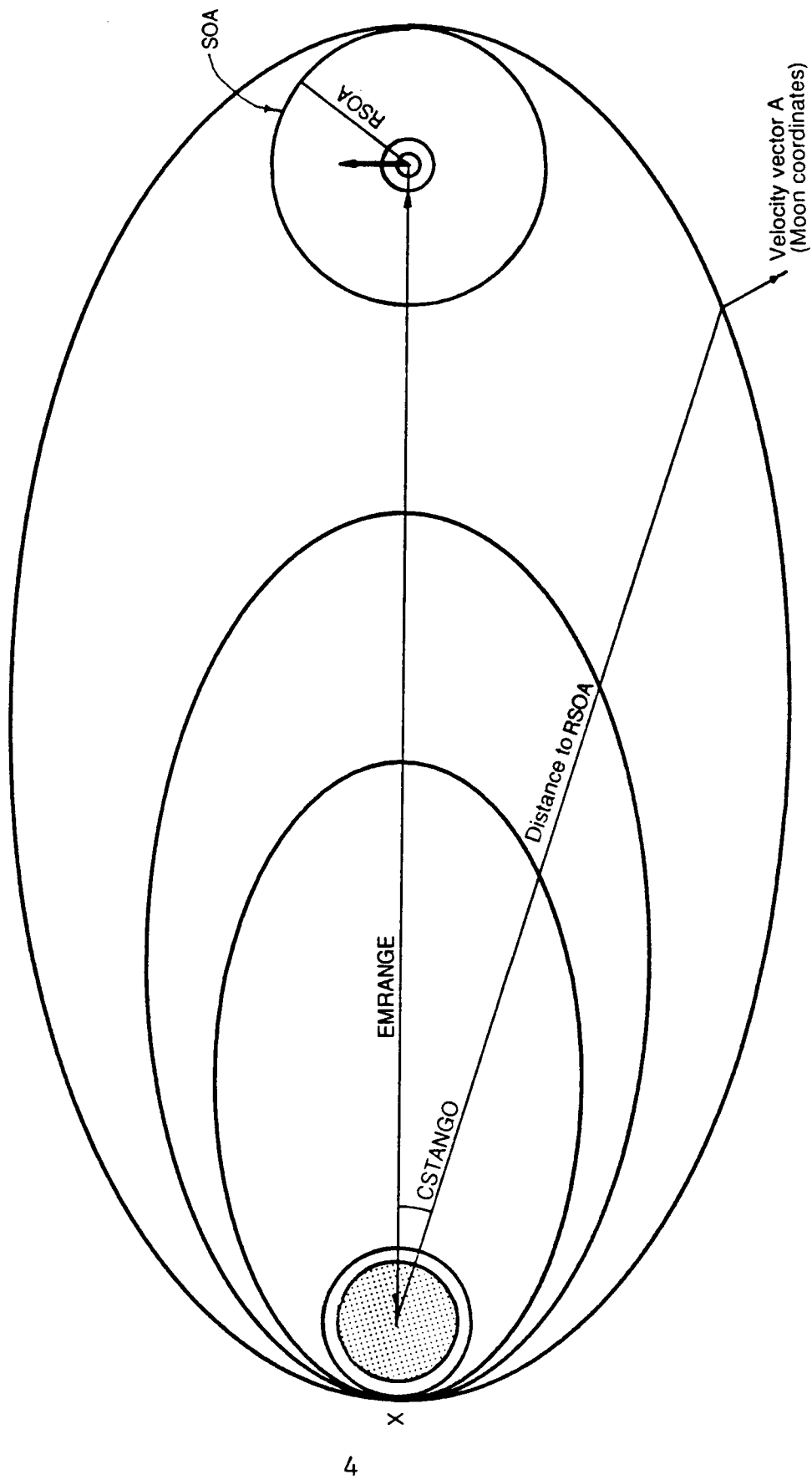


Figure 1

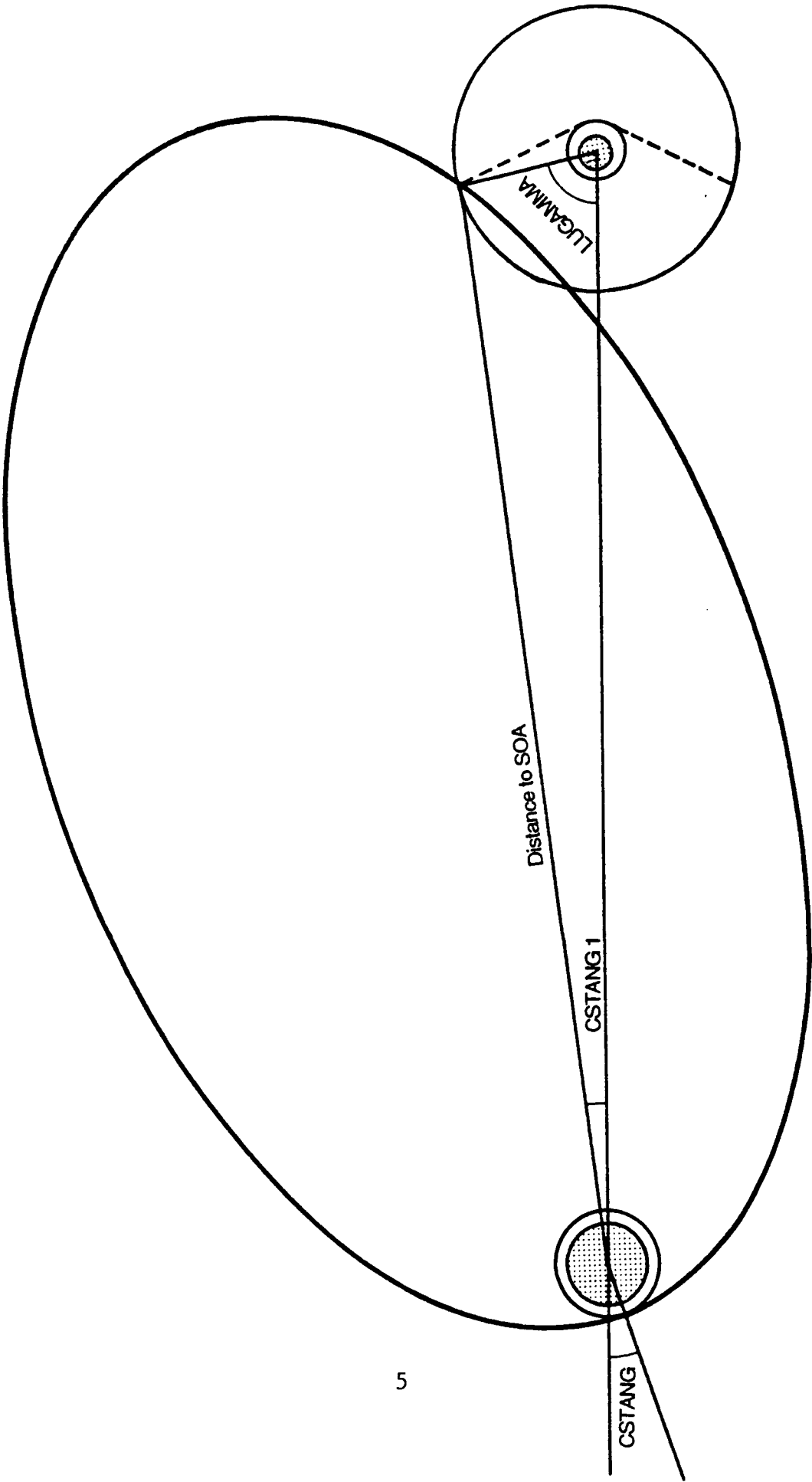
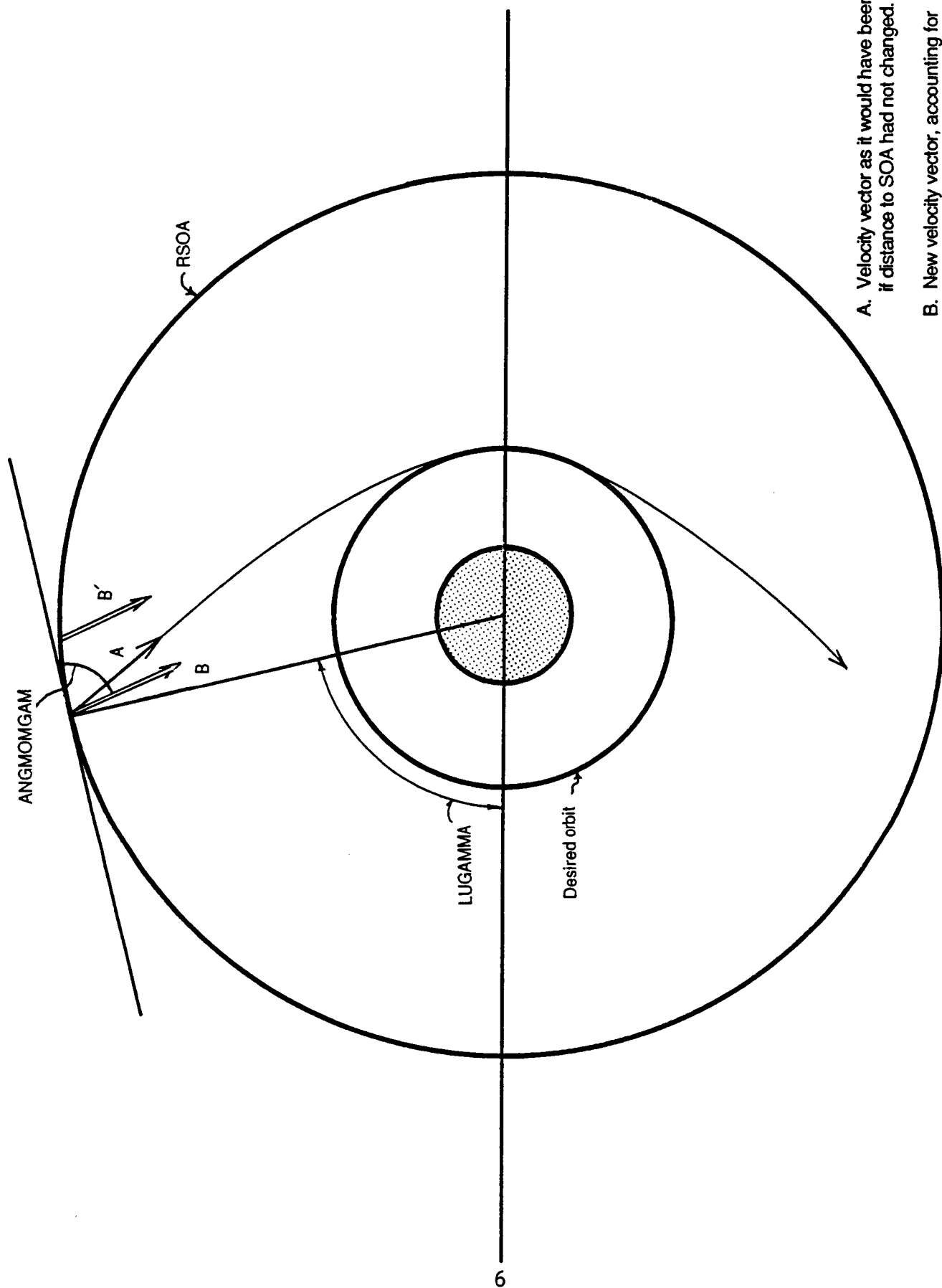


Figure 2



A. Velocity vector as it would have been if distance to SOA had not changed.

B. New velocity vector, accounting for increased distance to SOA.

B'. Desired position of new velocity vector, to intercept orbit perigee.

Figure 3



Figure 4

4.0 Program Execution Instructions

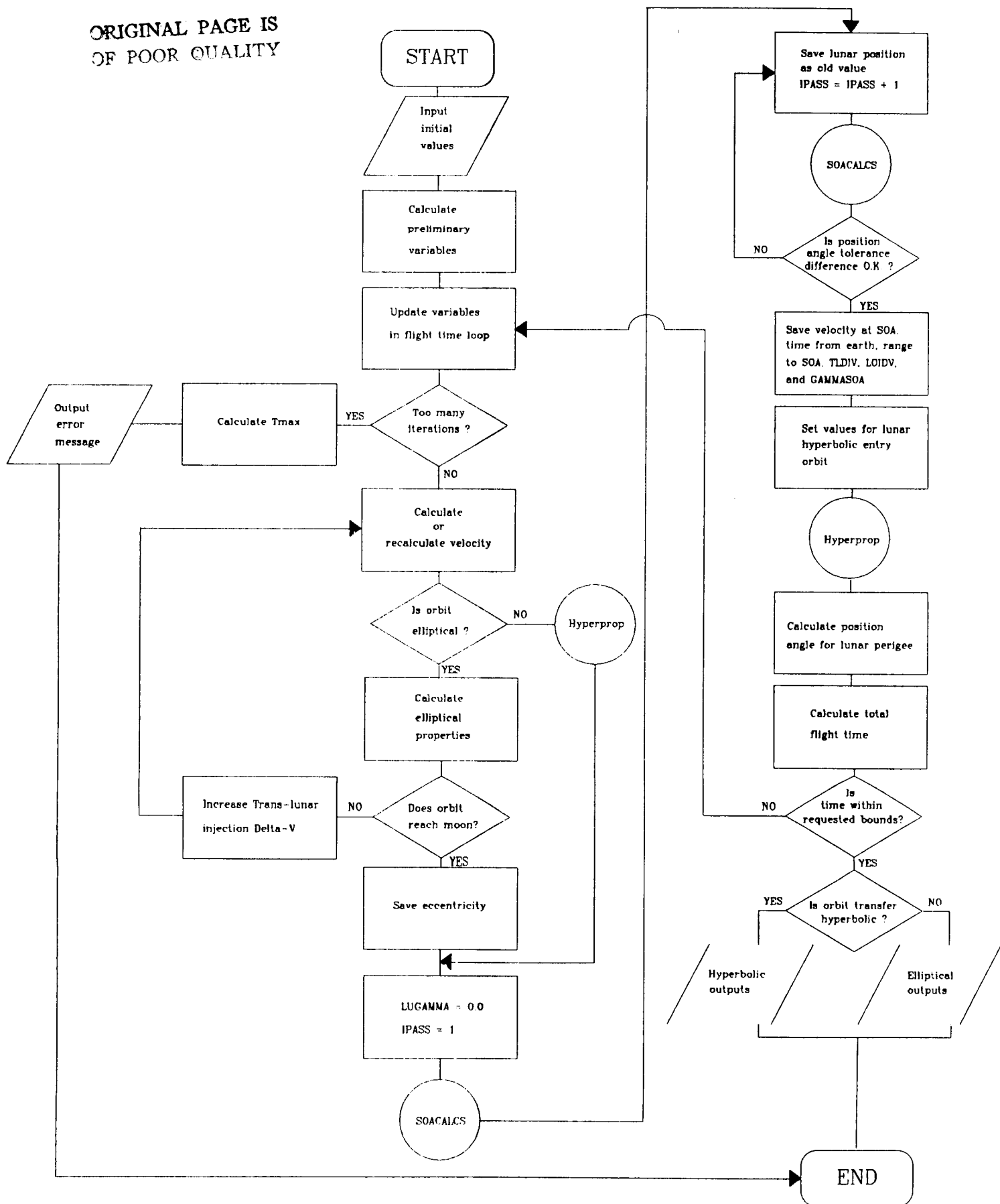
The following instructions describe the steps to be taken by the user to execute this program.

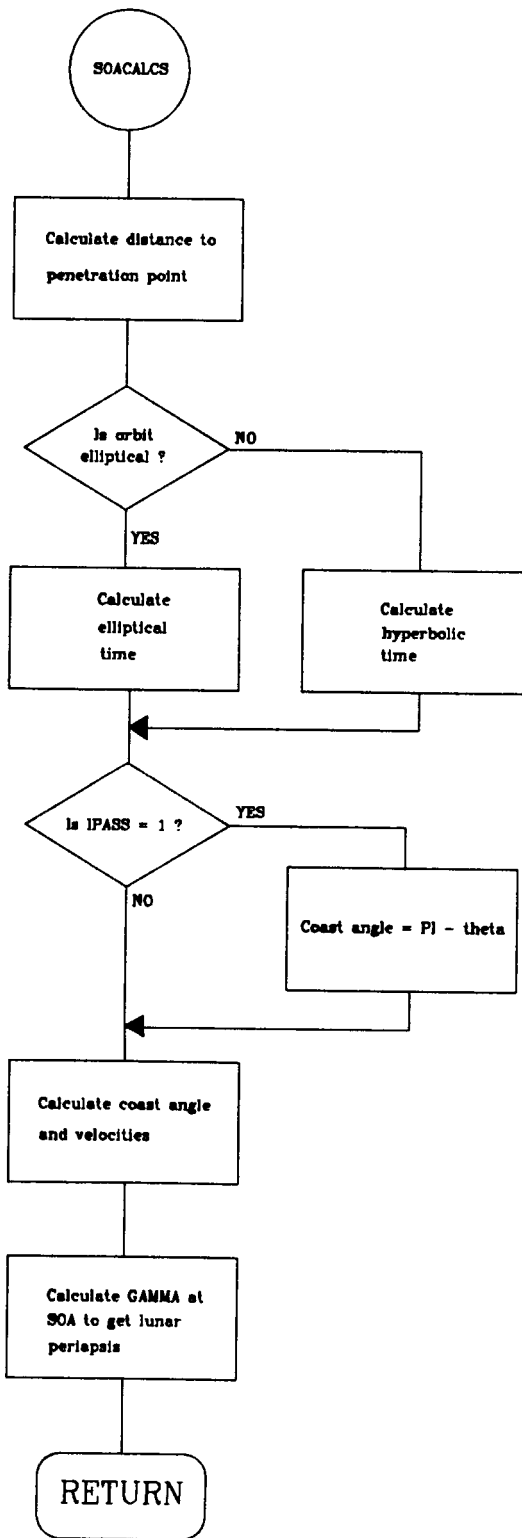
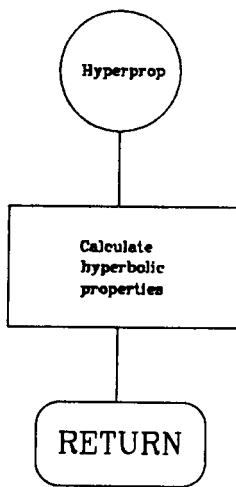
- A. Obtain access to the DEC VAX minicomputer and sign on with user identification.
- B. At the \$ prompt, type SETDRV 415 if using a Techtronix 4115 terminal, or SETDRV 407 if using a Techtronix 4111 terminal. If a terminal different from these is being used, consult the system administrator for the correct device driver code.
- C. At the next \$ prompt, type SETTEK LLOFX if using a Techtronix terminal, or RUN LLOFX if using a different terminal.
- D. When prompted by the program, enter the following information:
 1. Earth-orbiting space station altitude (kilometers above the Earth's surface).
 2. Desired orbital altitude above the Lunar surface (kilometers).
 3. Earth-Moon distance at time of Lunar intercept (kilometers). This can range from 359,856 to 405,970 kilometers. The average distance is 384,400 kilometers.
 4. Beginning reference point for a screen plot of the transfer orbit (kilometers). The Earth appears at the left side of the screen, and is centered at zero kilometers. The Moon appears at the right side of the screen, and is centered at the range specified in (3) above. The beginning reference point defines the left boundary of the plot. A negative beginning reference point ensures the entire Earth orbit is included in the plot.
 5. Ending reference point for the screen plot of the transfer orbit (kilometers). This defines the right boundary of the plot. It must be at least as large as the range specified in (3) above if the Moon is to be included in the plot.
 6. Aspect ratio (Y:X ratio) of the screen during the plot of the transfer orbit. This allows adjustment for the fact that a screen pixel is longer than it is wide, and removes the resulting distortion. The valid range for aspect ratio is 0.55 to 0.75.
- E. After the aspect ratio has been entered, the program will execute. The following information is displayed upon completion of execution:
 1. A message describing whether the transfer orbit is hyperbolic or elliptical.
 2. Trans-Lunar injection delta-v.
 3. Lunar orbit insertion delta-v.
 4. Trans-Earth injection delta-v.

5. Earth orbit insertion delta-v.
 6. Total delta-v.
 7. Actual time of flight for the outbound leg (Earth to Moon). Return time is the same.
 8. A screen plot of the transfer orbit.
- F. To re-execute the program with new parameters, begin again at step (C) above.

Appendix A - Program Flow Chart

ORIGINAL PAGE IS
OF POOR QUALITY





Appendix B - Program Code Listing

PROGRAM
THIS PROGRAM PROPAGATES ORBIT TO MOON AND BACK
IN METRIC VECTOR UNITS

PROGRAM WAS:

PROPOSED BY GUS BABB
WRITTEN BY CHRIS VARNER AND MIKE D'ONOFRIO
DOCUMENTED BY STEVE ERICKSON
FOR NASA'S ADVANCED SPACE TRANSPORTATION SYSTEM
CONTRACT NO. NAS 9-17878
EAGLE ENGINEERING INC. 1988

FIRST DECLARE VARIABLES REAL EXCEPT VARIABLES
BEGINNING WITH 'I'

IMPLICIT REAL*4 (A-H,J-Z)
REAL*4 X(250),Y(250),XE(250),YE(250),XL(250),YL(250)

OPEN GRAPHICS SUBROUTINES

CALL JBEGIN
CALL JDINIT(1)
CALL JDEVON(1)
CALL JIENAB(1,4,1)

INPUTS

WRITE (5,5)
FORMAT ('1INPUT ALL NUMBERS AS REAL VALUES')
WRITE (5,10)
FORMAT ('0INPUT SPACE STATION ALTITUDE km')
READ *,H
WRITE (5,15)
FORMAT ('0INPUT LUNAR ALTITUDE km')
READ *,HL
WRITE (5,17)
FORMAT ('0INPUT EARTH-MOON DISTANCE avg.=384400')
READ *,EMRANGE
WRITE (5,20)
FORMAT ('0INPUT FLIGHT TIME IN HOURS')
READ *,OFTHR WRITE (5,23)
FORMAT ('0INPUT END X POINT TO VIEW',/, ' FROM 0 TO 500000')
READ *,XMAX
WRITE (5,24)
FORMAT ('0INPUT ASPECT RATIO OF SCREEN (Y/X)',/, ' FROM
+ .55 TO .75')
READ *,AR
XMAX1=.2*(XMAX-XMIN)+XMAX
XMIN1=XMIN-.2*(ABS(XMAX-XMIN))
YMIN=(XMIN-XMAX)/2.0
YMAX=(XMAX-XMIN)/2.0
YMIN1=(XMIN1-XMAX1)/2.0
YMAX1=(XMAX1-XMIN1)/2.0

FUDGE FACTOR COMPENSATING FOR ASPECT RATIO OF SCREEN

YMAX=AR*YMAX
YMAX1=YMAX1*AR
ECCE=0.0
OBTALC=10000000.0
TIMEERROR=3601.0
TLIDV=3.07

FUDGE FACTOR COMPENSATING FOR ASPECT RATIO OF SCREEN[B

OBFLTTM=OFTHR*3600.
RTNFLTTM=RFTHR*3600.
GAMAIN =0.0
PI=3.1415926535
RO=6378.1
ROL = 1738.0
MUE=398603.0
MUL=4902.97
R=H+RO
VOM=1.02
RPE=R
V=SQRT(MUE/RPE)+TLIDV
ICOUNT=0
RSOA=EMRANGE/10.017
LOOPFLAG\$='ON'

START THE FLIGHT TIME LOOP

DO 1400 WHILE(LOOPFLAG\$.EQ.'ON')
 GAIN=1./(OBTALC*10.0)
 TLIDV=TLIDV+TIMEERROR*GAIN
 MU=MUE
 ICOUNT=ICOUNT +1
 R=H+RO
 RPE=R
 RPL=ROL+HL
 GAMMA=PI*GAMAIN/180.

IS THE ORBIT ELLIPTICAL OR HYPERBOLIC?

IF(ICOUNT.LT.10000)THEN
 V=SQRT(MUE/RPE)+TLIDV
 IF(V**2.0.LE.(2.*MU/R)) THEN

ORBIT IS ELLIPTICAL

Q=R*V**2./MU
RP=(1.-SQRT(1.-Q*(2.-Q)*(COS(GAMMA))**2)) *R/(2.-Q)
RA=(1.+SQRT(1.-Q*(2.-Q)*(COS(GAMMA))**2)) *R/(2.-Q)
A=(RA+RP)/2.
ECC=(RA-RP)/(RA+RP)
P=(1.+ECC)*RP
NU=SQRT(MU/A**3)

```

COFO=(P/R-1.)/ECC
FO=ATAN(SQRT(1.0001-COFO**2)/COFO)
IF (FO.LT.0.) FO=PI+FO
IF (GAMMA.LT.0.) FO=-FO
EO=2.*(ATAN(SQRT((1.-ECC)/(1.+ECC))*TAN(FO/2.)))
THETAO=FO
TAUO=(EO-ECC*SIN(EO))/NU
TIMEO=TAUO

```

DOES THE ELLIPSE REACH THE MOON?

```
IF (RA.LT.(EMRANGE+RSOA)) THEN
```

```
NO
```

```

    TLIDV=TLIDV + .01
    GO TO 250

```

```
ENDIF
```

```
ELSE
```

ORBIT IS HYPERBOLIC

CALCULATE HYPERBOLIC PROPERTIES

```
CALL HYPERPROP(PI,A,B,MU,R,V,GAMMA,P,ECC,RP,
    VP,COTHETA,THETAO,TIMEO)
```

```
ENDIF
```

```
LUGAMMA=0.0
```

```
IPASS=1
```

```

CALL SOACALCS(PI,VXL,IPASS,VYL,VT,GAMMAT,VPL,NU,
ANGMOMGAM,RSOA,TFROMPER,RT,MUL,RPL,ECC,P,MU,A,VOM,
LUGAMMA,LUGAMMANEW,EMRANGE,TAUO,CSTANG)

```

ITERATE

```
IPASS=IPASS+1
```

SAVE THE LUNAR POSITION ANGLE AS AN OLD VALUE

```
LUGAMMA=LUGAMMANEW
```

```

CALL SOACALCS(PI,VXL,IPASS,VYL,VT,GAMMAT,VPL,NU,
ANGMOMGAM,RSOA,TFROMPER,RT,MUL,RPL,ECC,P,MU,A,VOM,
LUGAMMA,LUGAMMANEW,EMRANGE,TAUO,CSTANG)
IF (ABS(LUGAMMANEW-LUGAMMA).GE..02) GOTO 200

```

THEN DO ANOTHER ITERATION

SAVE VELOCITY AT SOA, TIME FROM EARTH TO SOA, RANGE TO SOA
TLIDV, AND LOIDV, AND GAMMASOA AS OLD VALUES

```
TTSOA=TFROMPER
```

```
RESOA=RT
```

```
TLIDV=V-SQRT(MUE/(H+RO))
```

```
LOIDV=VPL-SQRT(MUL/RPL)
```

```
AE=A
```

NUE=NU
PE=P
ECCE=ECC
VESOA=VT
GAMMASOA=GAMMAT

SET VALUES FOR LUNAR HYPERBOLIC ENTRY ORBIT
MU=MUL
V=SQRT(VYL**2+VXL**2)
GAMMA=ANGMOMGAM
R=RSOA
CALL HYPERPROP(PI,A,B,MU,R,V,GAMMA,P,ECC,RP,VP,
COTHETA,THETAO,TIMEO)

CALCULATE THE POSITION ANGLE FOR LUNAR PERIGEE
THETALP=LUGAMMA+THETAO
CALCULATE THE TOTAL FLIGHT TIME AND
TIME INSIDE SOA
TIMESOATLOI=TIMEO
OBTALC= (ABS(TTSOA)+ABS(TIMESOATLOI))

CHECK TO SEE IF TIME IS WITHIN REQUESTED BOUNDS
TIMEERROR=OBTALC-OBFLTTM
IF (ABS(TIMEERROR).LE.600.) THEN
 OUTPUTS
 IS IT HYPERBOLIC?
 IF (ECCE.GE.1.0) THEN
 YES
 WRITE(5,50)
 FORMAT('-----HYPERBOLIC ORBIT-----')
 ELSE
 WRITE(5,55)
 FORMAT('-----ELLIPTICAL ORBIT-----')
 ENDIF
 WRITE(5,60) TLIDV,LOIDV
 FORMAT('00OUTBOUND PROPERTIES',/, ' TLI DELTA-V, km/sec.=',
F6.2,/, ' LOI DELTA-V, km/sec.= ',F6.2)
 WRITE(5,62) LOIDV,TLIDV
 FORMAT(' INBOUND PROPERTIES',/, ' TEI DELTA-V= ',F6.2,/,
 ' EOI DELTA-V = ',F6.2)
 WRITE(5,65) TLIDV+LOIDV,(ABS(TTSOA)+ABS(TIMESOATLOI))/3600.
 FORMAT(' TOTAL DELTA-V = ',F7.2,/, ' FLIGHT TIME = ',F10.3)
 ECCM=ECC

GRAPHICS

IT=200
ECC=ECCE
A=AE
P=PE
MU=MUE
NU=NUE
RI=6371.23
CALL GATTRI (1,0,1.0)

```

CALL GATTRI (2,0,1.0)
CALL GATTRI (3,0,1.0)
CALL GATTRI (4,5,1.0)
CALL GATTRI (5,5,1.0)
CALL GATTRI (6,5,1.0)
CALL GATTRI (7,5,1.0)
CALL GATTRI (11,5,1.0)
CALL GATTRI (12,5,1.0)
CALL GCHART (1,5,'EARTH-MOON ORBIT',16)
CALL GAXIS (1,0,XMIN1,XMAX1,0,
'DISTANCE WITH RESPECT TO EARTH CENTER',
37,YMIN1,YMAX1,0,'KM',2)
DO 1209 IE=1,3
  ICO=0
  DO 1009 IN=1,IT
    INM1=IN-1
    X(IN)=RI*COS(FLOAT(INM1)*10.*PI/180.)
    Y(IN)=RI*SIN(FLOAT(INM1)*10.*PI/180.)
    IF(IE.NE.1) X(IN)=X(IN)+EMRANGE
    IF(X(IN).GT.XMIN.AND.X(IN).LT.XMAX) THEN
      ICO=ICO+1
      Y(ICO)=Y(IN)
      X(ICO)=X(IN)
    ENDIF
  CONTINUE

  GRAPH

```

```

CALL JOPEN
CALL JCOLOR(6)
IF(IE.EQ.2) CALL JCOLOR(1)
IF(IE.EQ.3) THEN
  CALL JCOLOR(0)
  CALL GCURVE(X,Y,ICO,-1,0,0)
ENDIF
IF(IE.NE.3) CALL GCURVE(X,Y,ICO,0,0,0)
CALL JCLOSE
IF(IE.EQ.1) RI=1739.35
IF(IE.EQ.2) RI=RSOA
CONTINUE
ITIMESE=200
IXE=0
DO 1309 I=0,ITIMESE
  IP1=I+1
  RT=(RO+H+1.0)+FLOAT(I)*(RESOA-RO-H)/FLOAT(ITIMESE)
  CALL ORBITPROP(ECC,RT,P,PI,MU,A,THETA,TFROMPER,
    TAUO,GAMMAT,NU,VT)
  XE(IP1)=RT*(-1.0)*COS(CSTANG+THETA)
  YE(IP1)=RT*(-1.0)*SIN(CSTANG+THETA)
  IF (XE(IP1).GT.XMIN.AND.XE(IP1).LT.XMAX) THEN
    IXE=IXE+1
    XE(IXE)=XE(IP1)
    YE(IXE)=YE(IP1)
  ENDIF

```

```

309      CONTINUE
        CALL JOPEN
        CALL JCOLOR(2)
        CALL GCURVE(XE,YE,IXE,0,0,0)
        CALL JCLOSE
        ECC=ECCM
        MU=MUL
        V=SQRT(VYL**2+VXL**2)
        GAMMA=ANGMOMGAM
        R=RSOA
        CALL HYPERPROP(PI,A,B,MU,R,V,GAMMA,P,ECC,RP,
+         VP,COTHETA,THETAO,TIMEO)
        THETALP=LUGAMMA+THETAO
        TIMESOATLOI=TIMEO
        ITIMES=31
        ITIMESM1=ITIMES -1
        IXL=0
        DO 1359 I=0,ITIMESM1
            IP1=I+1
            RT=RSOA-FLOAT(I)*(RSOA-RP)/(FLOAT(ITIMESM1))
            CALL ORBITPROP(ECC,RT,P,PI,MU,A,THETA,TFROMPER,
+            TAUO,GAMMAT,NU,VT)
            XL(IP1)=EMRANGE-RT*COS(THETALP-THETA)
            YL(IP1)=RT*SIN(THETALP-THETA)
            IF (XL(IP1).GT.XMIN.AND.XL(IP1).LT.XMAX) THEN
                IXL=IXL+1
                XL(IXL)=XL(IP1)
                YL(IXL)=YL(IP1)
            ENDIF
359      CONTINUE
        CALL JOPEN
        CALL JCOLOR(4)
        CALL GCURVE(XL,YL,IXL,0,0,0)
        CALL JCLOSE
        INBOUND
        DO 1379 I=1,IXE
            YE(I)=YE(I)*(-1.0)
379      CONTINUE
        DO 1389 I=1,IXL
            YL(I)=YL(I)*(-1.0)
389      CONTINUE
        CALL JOPEN
        CALL JCOLOR(2)
        CALL GCURVE(XE,YE,IXE,0,0,0)
        CALL JCOLOR(4)
        CALL GCURVE(XL,YL,IXL,0,0,0)
        CALL JCLOSE
        LOOPFLAG$='OFF'
        ENDIF
    ELSE
        PRINT *,'*****LOOPING ERROR OCCURED*****'
        PRINT *,'PROBABLY CAUSED BY EXCESSIVE FLIGHT TIME'
        PRINT *,'MAXIMUM FLIGHT TIME IS APPROX 75 HRS AT LUNAR'
        PRINT *,'PERIGEE AND 120 HRS AT LUNAR APOGEE'

```

```

        LOOPFLAG$='OFF'
    ENDIF
)0 CONTINUE
    CALL JPAUSE(1)
    CALL JDEVOF(1)
    CALL JDEND(1)
    CALL JEND
    STOP
    END

```

```

+ SUBROUTINE HYPERPROP (PI,A,B,MU,R,V,GAMMA,P,ECC,RP,
    VP,COTHETA,THETAO,TIMEO)

```

```

    IMPLICIT REAL*4 (A-Z)
    A=MU*R/(R*V**2-2.0*MU)
    B=SQRT(R**3*V**2*(COS(GAMMA))**2/(R*V**2-2.0*MU))
    P=R**2*V**2*COS(GAMMA)**2/MU
    ECC=SQRT(P/A+1.0)
    RP=A*(ECC-1.0)
    VP=SQRT(MU/P)*(1.0+ECC)
    COTHETA=(P/R-1.0)/ECC
    THETAO=ATAN(SQRT(1.0001-COTHETA**2)/COTHETA)
    IF(THETAO.LT.0.0) THETAO=PI+THETAO
    IF(GAMMA.LT.0.0) THETAO=-THETAO
    TIMEO1=SQRT(P**3/MU)/(ECC**2-1.0)
    TIMEO2=ECC*SIN(THETAO)/(1.0+ECC*COTHETA)
    TIMEO3=(1.0/SQRT(ECC**2-1.0))*LOG((ECC+COTHETA+SQRT(
+     ECC**2-1.0)*SIN(THETAO))/(1.0+ECC*COTHETA))
    TIMEO=TIMEO1*(TIMEO2-TIMEO3)
    RETURN
    END

```

```

+ SUBROUTINE SOACALCS (PI,VXL,IPASS,VYL,VT,GAMMAT,VPL,NU,
+     ANGMOMGAM,RSOA,TFROMPER,RT,MUL,RPL,ECC,
+     P,MU,A,VOM,LUGAMMA,LUGAMMANEW,EMRANGE,
+     TAUO,CSTANG)

```

```

    IMPLICIT REAL*4 (A-H,J-Z)
    DL=RSOA*COS(LUGAMMA)
    DE=EMRANGE-DL
    RT=SQRT(DE**2+RSOA**2-DL**2)
    CALL ORBITPROP (ECC,RT,P,PI,MU,A,THETA,TFROMPER,
+     TAUO,GAMMAT,NU,VT)
    RY=RSOA*SIN(LUGAMMA)
    CSTANG1=ATAN(RY/DE)
    CSTANG=PI+CSTANG1-THETA
    VXE=VT*SIN(GAMMAT+PI-CSTANG-THETA)
    VYE=VT*COS(GAMMAT+PI-CSTANG-THETA)

```

```
VXL=VXE
VYL=VYE-VOM
```

```
CALCULATE GAMMA AT SOA TO GET LUNAR PERIAPSIS
```

```
VSOAL2=VXL**2 + VYL**2
VPL2= VSOAL2-(2.0*MUL/RSOA)+(2.0*MUL/RPL)
VSOAL=SQRT(VSOAL2)
VPL=SQRT(VPL2)
ANGMOMGAM=ATAN(SQRT(VSOAL2*RSOA**2-VPL2*RPL**2)/
(VPL*RPL))
LUGAMMANEW=ATAN(-VYL/VXL)-ANGMOMGAM +PI/2.
RETURN
END
```

```
SUBROUTINE ORBITPROP(ECC,RT,P,PI,MU,A,THETA,TFROMPER,
TAUO,GAMMAT,NU,VT)
```

```
IMPLICIT REAL*4(A-H,J-Z)
IF(ECC.LT.1.0) THEN
```

```
    ELLIPTICAL ORBIT
    PROPAGATE ELLIPTICAL ORBIT FORWARD TO RT OUTBOUND
```

```
    F=ATAN((SQRT(ECC**2*RT**2-(P-RT)**2))/(P-RT))
    IF(F.LT.0.0) F=F+PI
    VT=SQRT(MU*(2.0/RT-1.0/A))
    GAMMAT=ATAN(ECC*SIN(F)/(1.0+ECC*COS(F)))
    ETH=2.0*(ATAN(SQRT((1.0-ECC)/(1.0+ECC))*TAN(F/2.0)))
    TAUTH=(ETH-ECC*SIN(ETH))/NU
    TFROMPER=TAUTH-TAUO
    THETA=F
```

```
ELSE
```

```
    HYPERBOLIC ORBIT
    ORBIT PROPAGATION FOR HYPERBOLIC ORBIT
```

```
    COTHETA=(P/RT-1.0)/ECC
    THETA=ATAN(SQRT(1.0001-COTHETA**2)/COTHETA)
    IF(THETA.LT.0.0) THETA=PI+THETA
    VT=SQRT(MU*(2.0/RT+1.0/A))
    COGAMAT=SQRT(P*MU)/(RT*VT)
    GAMMAT=ATAN(SQRT(1.0001-COGAMAT**2)/COGAMAT)
    IF(THETA.LT.0.0) GAMMAT=-GAMMAT
    TFROMPER1=SQRT(P**3/MU)/(ECC**2-1.0)
    TFROMPER2=ECC*SIN(THETA)/(1.0+ECC*COTHETA)-(1.0/SQRT(
ECC**2-1.0))*LOG((ECC+COTHETA+SQRT(ECC**2-1.0)
*SIN(THETA))/(1.0+ECC*COTHETA))
```

```
    TFROMPER=TFROMPER1*TFROMPER2
```

```
ENDIF
RETURN
END
```


Appendix C - Program Variables

<u>VARIABLE</u>	<u>DESCRIPTION</u>
A	Semimajor Axis of the Transfer Orbit (km)
AE	Semimajor Axis of the Transfer Orbit (km), Earth coordinates. Stored Value of "A" Before Switching to Lunar Coordinates.
AR	Aspect Ratio of Screen Plot
ANGMOMGAM	Flight Path Angle at Sphere of Action Penetration Point for the Lunar Hyperbolic Orbit (rad)
B	Hyperbolic Asymptotes Parameter
COFO	Cosine of the True Anomaly (dimensionless)
COGAMAT	Cosine of the Transfer Orbit Flight Path Angle at the SOA (non-dimensional)
COTHETA	Cosine of the True Anomaly for Hyperbolic Orbits (non-dimensional)
CSTANG	Actual Coast Angle (rad)
CSTANG1	Angle from the Earth-Moon Line to the Sphere of Action Penetration Point (rad)
CSTANGO	Coast Angle Initial Guess (rad)
DE	X Component of Distance from the Earth to Sphere of Action Penetration Point (km)
DL	X Component of Distance from the Moon to Sphere of Action Penetration Point, Measured Positive in the Negative X Direction (km)
ECC	"New" Eccentricity of the Transfer Orbit (dimensionless)
ECCE	"Old" Eccentricity of the Transfer Orbit (dimensionless)
ECCM	Eccentricity of the Hyperbolic Lunar Fly-By Orbit
EMRANGE	Earth-Moon Range (km)
EO	Initial Eccentric Anomaly Along the Transfer Orbit (rad)
ETH	Eccentric Anomaly for the Point of Sphere of Action Penetration Along the Transfer Orbit (rad)

F	True Anomaly for the Intersection Point of the Transfer Orbit and the Lunar Sphere of Action (rad)
FO	Initial True Anomaly Along the Transfer Orbit (rad)
GAIN	Error Multiplier for Adjusting Initial Velocity Change for a New Flight Time (delta-v/s)
GAMAIN	Inertial Flight Path Angle at Transfer Orbit Perigee (deg)
GAMMA	Inertial Flight Path Angle at Transfer Orbit Perigee (rad)
GAMMASOA	Final Inertial Flight Path Angle of the Transfer Orbit at the Sphere of Action Penetration Point (rad)
GAMMAT	Inertial Flight Path Angle of the Transfer Orbit at Sphere of Action Penetration (rad)
H	Space Station Orbital Altitude (km)
HL	Lunar Orbit Altitude (km)
I	Iteration Counter Used During Transfer and Lunar Fly-By Orbit Plots
ICO	Plot Matrix Subscript Used to Store Circle Plot Position Values Within the User-Specified Range
ICOUNT	Iteration Counter
IE	Identifier Used During Circle Plotting to Determine If Circle is the Earth, the Moon, or the SOA
IN	Iteration Counter Used During Circle Plotting Routines
INM1	IN Minus 1
IP1	I Plus 1
IPASS	Iteration Counter
IT	Constant. Maximum Value of IN
ITIMES	Number of Plot Positions in Lunar Fly-By Orbit
ITIMESE	Number of Plot Positions in Transfer Orbit
ITIMESM1	ITIMES Minus 1
IXE	Plot Matrix Subscript Used to Store Transfer Orbit Plot Position Values Within the User-Specified Range
IXL	Plot matrix Subscript Used to Store Hyperbolic Lunar Fly-By Orbit Plot Position Values Within the User-Specified Range

LOIDV	Lunar Orbit Injection Velocity Change (km/s)
LOOPFLAG	Do-While Flag That Limits the Number of Iterations Seeking Correct Flight Time
LUGAMMA	Angle Between the Earth-Moon Line and the "Old" Sphere of Action Penetration Point (rad)
LUGAMMANEW	Angle Between the Earth-Moon Line and the "New" Sphere of Action Penetration Point (rad)
MU	Local Gravity Constant (km^3/s^2)
MUE	Gravitational Constant for the Earth (km^3/s^2)
MUL	Gravitational Constant for the Moon (km^3/s^2)
NU	Mean Motion of the Transfer Orbit (rad/s)
NUE	Stored Value of NU. Recalled During Plotting Routines
OBFLTTM	Outbound Flight Time (secs)
OBTALC	Total Time of Flight from Earth Orbit to Lunar Orbit (sec)
OFTHR	Outbound Flight Time (hrs)
P	Semi-Latus Rectum of the Transfer Orbit (km)
PE	Stored Value of P. Recalled During Plotting Routines
Q	Vis-Viva Parameter (dimensionless)
R	Orbital Radius of the Space Station (km)
RA	Radius to the Apogee of the Transfer Orbit (km)
RESOA	Final Distance to the Sphere of Action (km)
RFTHR	Return Flight Time (hrs)
RI	Various Circle Radii, Used During Plotting Routines
RO	Radius of the Earth (km)
ROL	Radius of the Moon (km)
RP	Radius to the Perigee of the Transfer Orbit (km)
RPE	Perigee Radius at the Earth (km)
RPL	Perigee Radius at the Moon (km)

RSOA	Radius of the Lunar Sphere of Action (km)
RT	Range to Lunar Sphere of Action (km)
RTNFLTMM	Return Flight Time (secs)
RY	Y Component of the Distance from the Moon to the Sphere of Action Penetration Point (km)
TAUO	Initial Time from Perigee (sec)
TAUTH	Time of Sphere of Action Penetration from Transfer Orbit Perigee (sec)
TFROMPER	Time of Sphere of Action Penetration from TLI (sec)
TFROMPER1	Intermediate Calculation for TFROMPER
TFROMPER2	Intermediate Calculation for TFROMPER
THETA	Position Angle for the Sphere of Action Penetration Point, Earth Reference (rad)
THETALP	Lunar Perigee Position Angle (rad)
THETAO	Initial Position Angle, and True Anomaly for Hyperbolic Orbits (rad)
TIMEERROR	Difference Between Total Time of Flight and Requested Time of Flight (sec)
TIMEO	Initial Time From Perigee (sec)
TIMEO1	Intermediate Calculation for TIMEO
TIMEO2	Intermediate Calculation for TIMEO
TIMEO3	Intermediate Calculation for TIMEO
TIMESOATLOI	Time from Sphere of Action Penetration to Lunar Orbit Injection (sec)
TLIDV	Trans Lunar Injection Velocity Change (km/s)
TMAX	Approximate Maximum Flight Time Allowed (hrs)
TTSOA	Final Time to the Sphere of Action (sec)
V	Velocity (km/s)
VESOA	Inertial Velocity at the Sphere of Action Penetration Point (km/s)
VOM	Velocity of the Moon (km/s)

VP	Hyperbolic Velocity at Perigee (km/s)
VPL	Lunar Relative Velocity at the Lunar Orbit Radius (km/s)
VPL2	Square of the Lunar Relative Velocity at the Lunar Orbit Radius (km²/s²)
VSOAL	Lunar Relative Velocity at the Sphere of Action (km/s)
VSOAL2	Square of the Lunar Relative Velocity at the Sphere of Action (km²/s²)
VT	Velocity at Sphere of Action Penetration (km/s)
VXE	X Velocity Component (Earth Coordinates: Inertial)
VXL	X Velocity Component (Lunar Coordinates)
VYE	Y Velocity Component (Earth Coordinates: Inertial)
VYL	X Velocity Component (Lunar Coordinates)
X	X-Component of Plot Position For Circles
XE	X-Component of Plot Position For Transfer Orbit
XL	X-Component of Plot Position For Hyperbolic Lunar Fly-By Orbit
XMAX	High X-Boundary Value Supplied By User For Screen Plot
XMAX1	Additional 20% Added to XMAX For Plot Border
XMIN	Low X-Boundary Value Supplied By User For Screen Plot
XMIN1	Additional 20% Subtracted From XMIN For Plot Border
Y	Y-Component of Plot Position For Circles
YE	Y-Component of Plot Position For Transfer Orbit
YL	Y-Component of Plot Position For Hyperbolic Lunar Fly-By Orbit
YMAX	High Y-Boundary Value For Screen Plot, Adjusted For Aspect Ratio
YMAX1	Preliminary High Y-Boundary Value For Screen Plot
YMIN	Low Y-Boundary Value For Screen Plot

Appendix D - Calculation of CSTANG

The coast angle (CSTANG) is the angle past the Earth-Moon line at which a vehicle performs a TLI burn, allowing it to intercept the Moon's SOA at the proper point (see Figure D-1).

The radial vector \vec{r}_A locates the point on the transfer ellipse such that this vector's magnitude equals the distance from the Earth to the SOA at the SOA's closest point. F0 is defined to be the true anomaly of \vec{r}_A (see Figure D-2).

The radial vector \vec{r}_B locates the SOA impact point. F1 is defined to be the true anomaly of \vec{r}_B . ΔF is defined to be the difference between the anomalies.

$$\Delta F = F1 - F0.$$

The initial coast angle (CSTANG0) is the angle between the major axis of the transfer ellipse and \vec{r}_A .

$$CSTANG0 = \pi - F0.$$

The angle of SOA impact (CSTANG1) is the angle between the Earth-Moon line and \vec{r}_B .

$$CSTANG1 = \tan^{-1} \left[\frac{RSOA * \sin(LUGAMMA)}{EMRANGE - RSOA * \cos(LUGAMMA)} \right]$$

From Figure D-1 it can be seen that

$$\begin{aligned} CSTANG &= CSTANG0 + CSTANG1 - \Delta F \\ &= (\pi - F0) + CSTANG1 - (F1 - F0) \\ &= \pi + CSTANG1 - F1. \end{aligned}$$

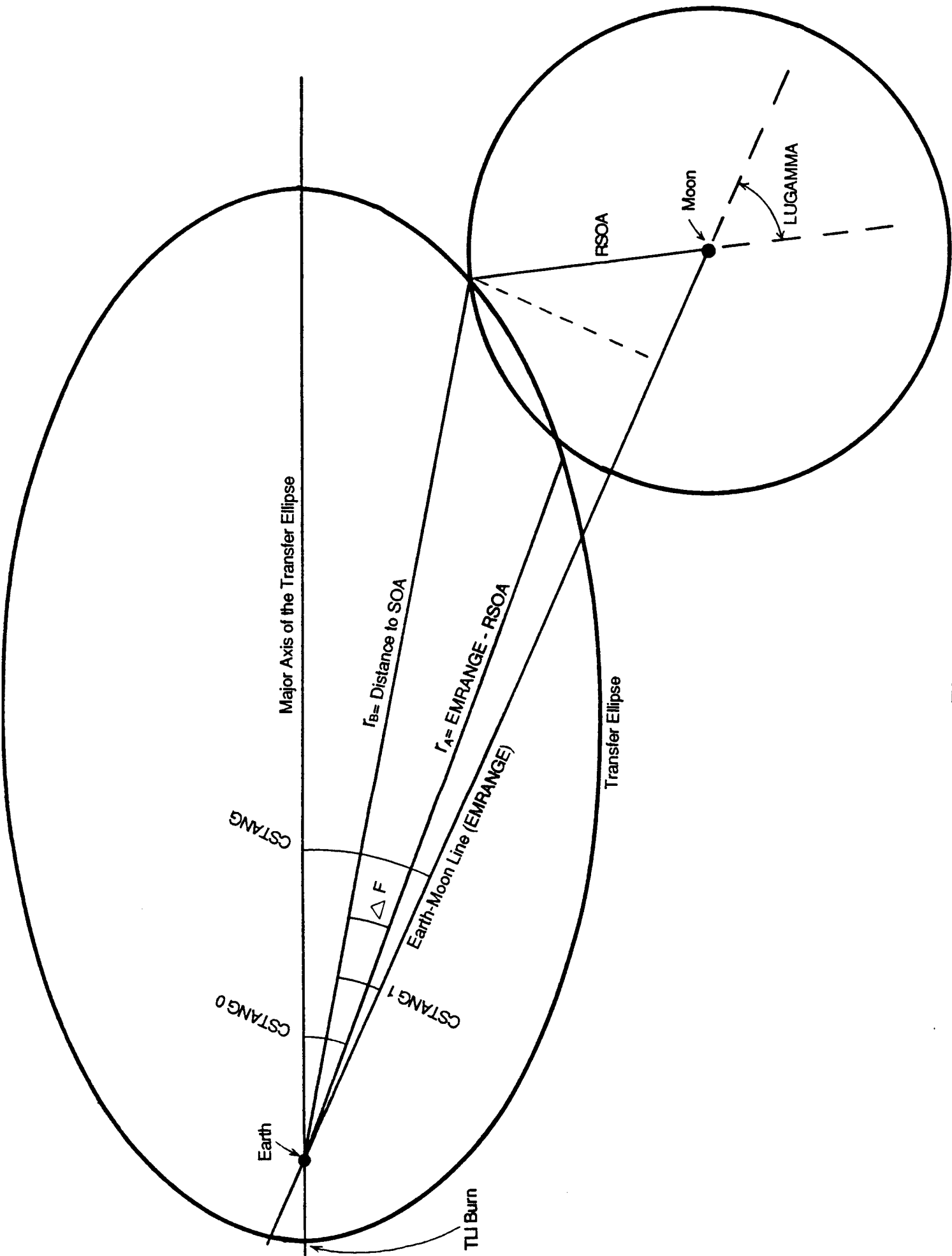


Figure D-1

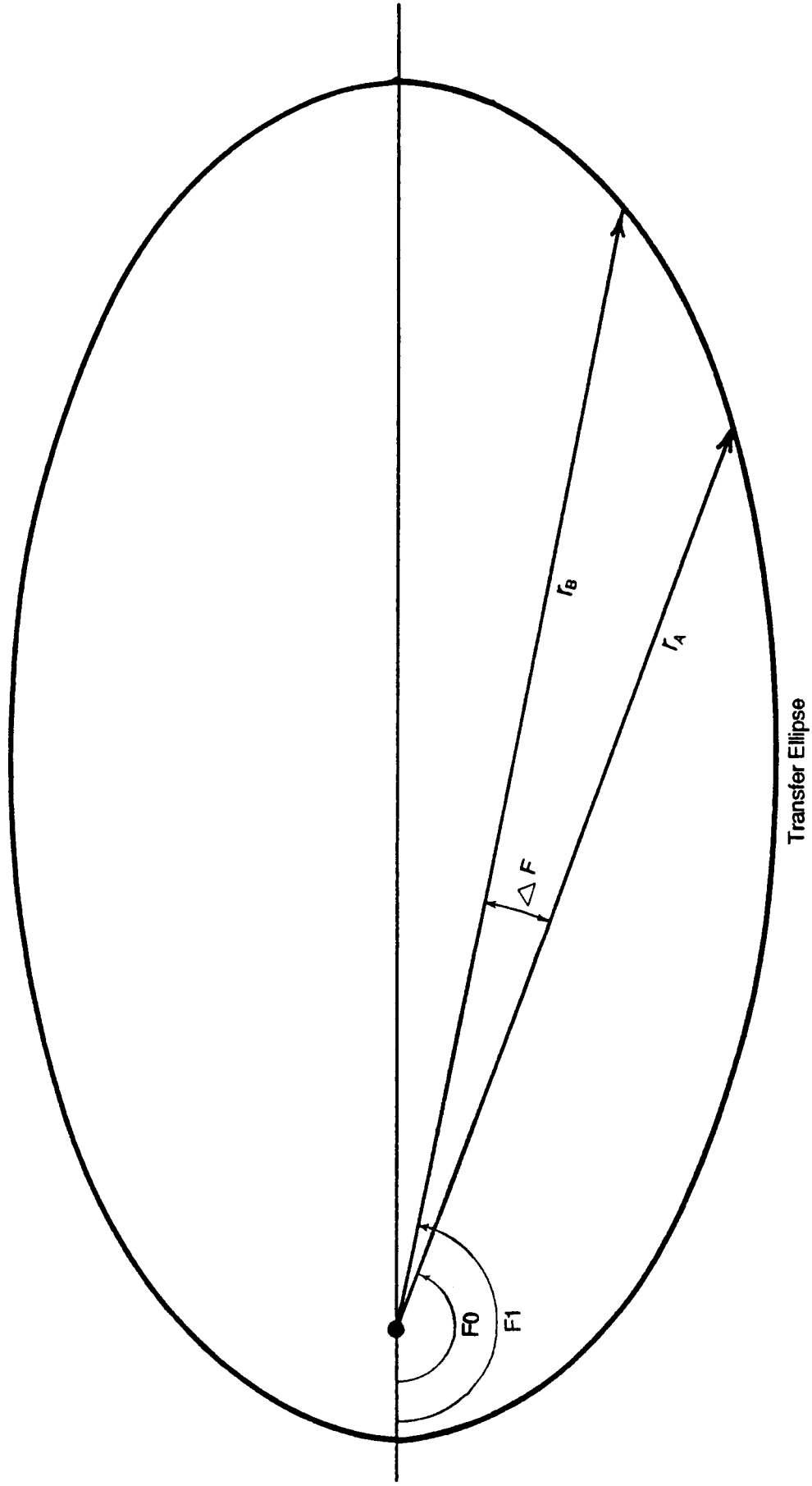


Figure D-2

Appendix E - Calculation of ANGMOMGAM

The flight path angle (γ) of the vehicle at Lunar SOA is calculated in terms of the vehicle's angular momentum (\vec{h}). The angular momentum is defined to be the cross-product of the position vector (\vec{r}) and the linear momentum vector ($m \vec{v}$), where γ is in the \vec{r}, \vec{v} plane and \vec{h} is perpendicular to this plane (see Figures E-1 and E-2).

$$\vec{h} = \vec{r} \times m \vec{v} = h \cdot \vec{u} = (m \cdot r \cdot v \cdot \cos \gamma) \cdot \vec{u}$$

$$h = m \cdot r \cdot v \cdot \cos \gamma$$

Angular momentum is constant along a given orbit. Therefore, the angular momentum at perigee is the same as the angular momentum at the sphere of action. If the magnitude of the velocity vector is known at the sphere of action, then the velocity at a specified perigee (v_p) can be calculated.

$$v_p = \sqrt{v^2 - v_{\text{esc(SOA)}}^2 + v_{\text{esc(perigee)}}^2}$$

$$= \sqrt{v^2 - 2 \cdot (\mu_M / r_{\text{SOA}}) + 2 \cdot (\mu_M / r_p)}$$

where v = magnitude of the velocity vector at SOA
 μ_M = moon's gravitational constant
 r_{SOA} = radial distance to the SOA
 r_p = radial distance to perigee.

At perigee, the flight path angle is zero by definition, and the magnitude of the orbit's angular momentum is the simple product of the distance and velocity.

$$h = m \cdot r_p \cdot v_p$$

Since the angular momentum is the same at the sphere of action, the flight path angle can be calculated.

$$h = m \cdot r_p \cdot v_p = m \cdot r_{SOA} \cdot v \cdot \cos \gamma$$

$$\gamma_{SOA} = \cos^{-1} \left[\frac{r_p \cdot v_p}{r_{SOA} \cdot v_{SOA}} \right] = \text{ANGMOMGAM.}$$

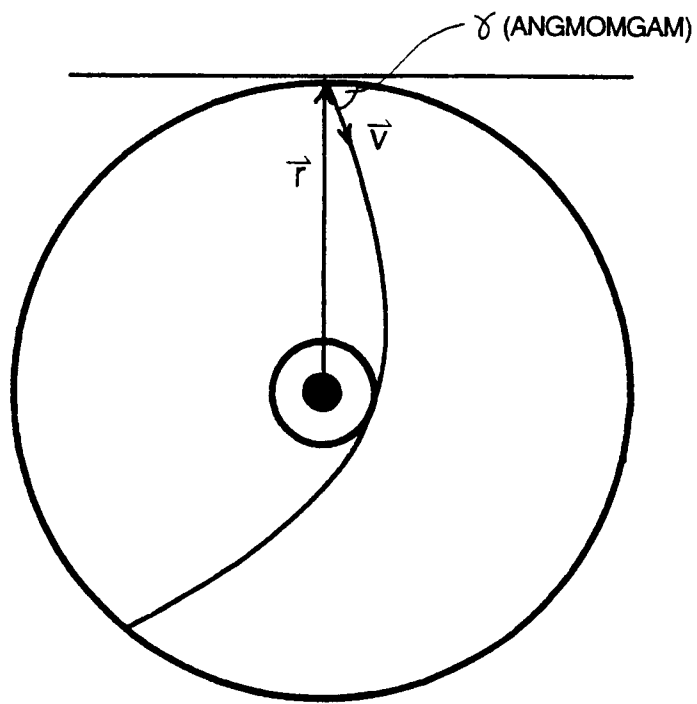


Figure E-1

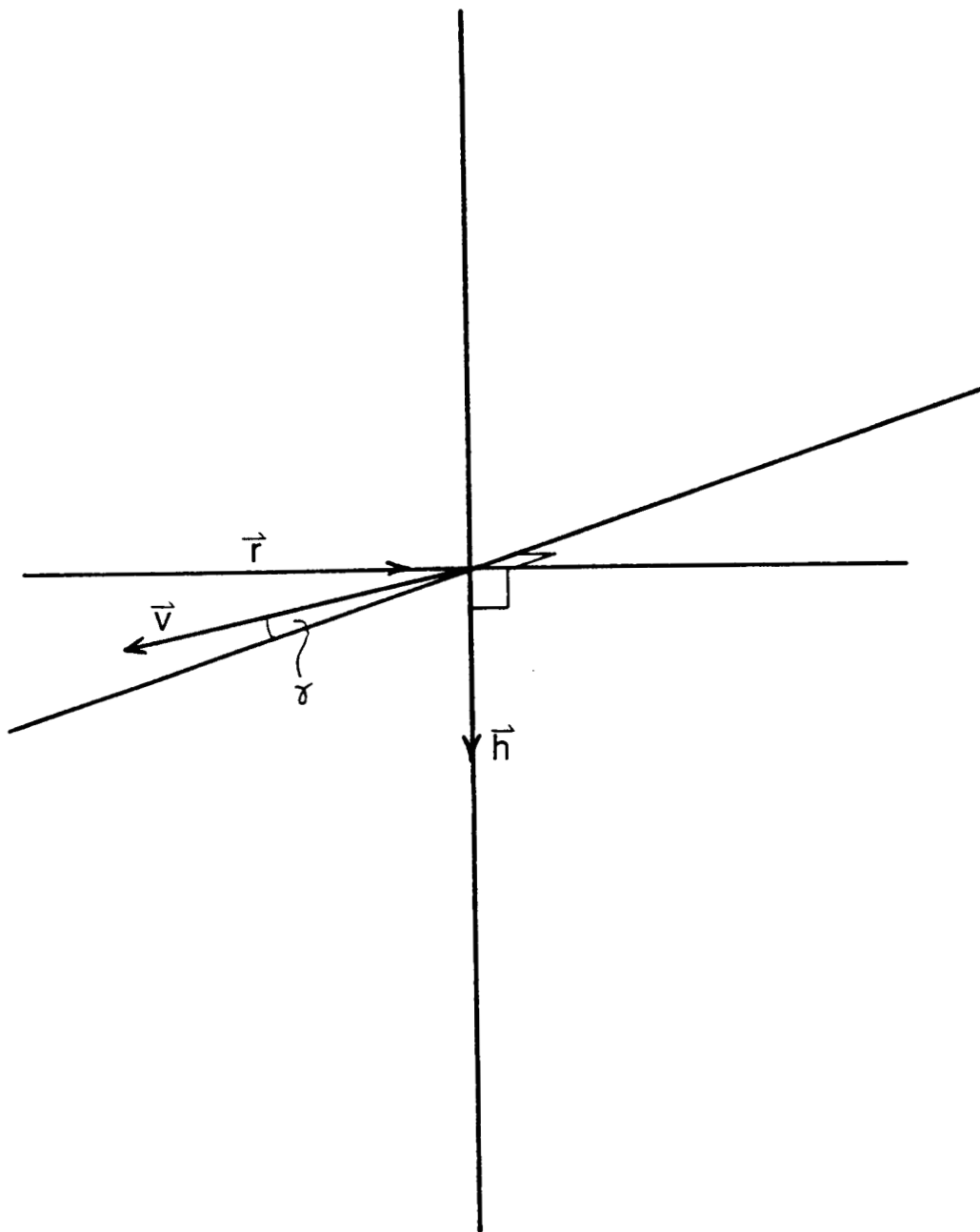


Figure E-2

Appendix F - Test Cases

The attached tables show actual Apollo transfer orbit data compared with the outputs of the program LLOFX. The burns shown in the chart include the Trans-Lunar Injection (TLI), Lunar Orbit Insertion (LOI), and Trans-Earth Injection (TEI). The TLI burn puts the vehicle in a hyperbolic or elliptical orbit to get in the sphere of influence about the moon. Next, the LOI burn puts the vehicle in a circular orbit of 60 nautical miles around the moon. When returning to the Earth a TEI burn is made in lunar holding orbit. The Apollo flights directly entered the Earth's atmosphere to perform a splash down. The program assumes that vehicles will come back to a transportation node in low Earth orbit (i.e. space station). To simulate an Apollo entry, the program LLOFX was run with an Earth holding orbit of 10.16 km. Listed are differences that may cause discrepancies: Apollo used free return trajectories that are not used in LLOFX. Apollo vehicles made plane changes going into transfer orbits and during Lunar Orbit Insertion, while LLOFX does not. Apollo confronted out-of-plane Earth-moon trajectories while LLOFX assumes in-plane. Apollo spacecraft had gravity losses due to long burns versus LLOFX's assumption of instantaneous burns. Also, Earth-moon perturbations might also affect output.

Although many assumptions are made, the numbers are within requested bounds. The TLI burns are less than 5% off the Apollo data and LOI numbers are within approximately 10% of the Apollo data. Therefore, LLOFX is useful for rough performance estimates for lunar vehicles.

OUTBOUND PROPERTIES

Check Cases

Flight #	Earth Moon Distance (km)	Flight Time (hrs)	Earth Orbit Altitude (km)	Lunar Orbit Altitude (km)	TLI Delta-V (km/sec)	LOI Delta-V (km/sec)
Apollo 11 LLOFX	351600 351600	73.09 73.02	334.7 334.7	111.2 111.2	3.18 3.10	.89 .81
Apollo 12 LLOFX	377900 377900	80.63 80.78	356.0 356.0	111.2 111.2	3.20 3.10	.88 .81
Apollo 14 LLOFX	372500 372500	79.47 79.21	331.9 331.9	107.7 107.7	3.16 3.11	.92 .81
Apollo 15 LLOFX	350900 350900	75.68 74.75	257.6 257.6	106.9 106.9	3.17 3.12	.91 .80
Apollo 16 LLOFX	371800 371800	76.00 76.09	316.9 316.9	108.4 108.4	3.17 3.11	.85 .82
Apollo 17 LLOFX	398900 398900	83.00 82.89	300.9 300.9	108.8 108.8	3.12 3.12	.79 .83

INBOUND PROPERTIES

Check Cases

Flight #	Flight Time (hrs)	Earth Altitude (km)	TEI Delta-V (km/sec)
Apollo 11 LLOFX	59.66 59.72	10.2 10.2	1.00 .90
Apollo 12 LLOFX	71.88 71.88	10.2 10.2	.93 .86
Apollo 14 LLOFX Ref. 1	63.22 63.30	10.2 10.2	1.06 .88 .93
Apollo 15 LLOFX	71.17 71.25	10.2 10.2	.93 .81
Apollo 16 LLOFX Ref. 1	65.27 65.31	10.2 10.2	1.03 .89 .93
Apollo 17 LLOFX	67.61 67.68	10.2 10.2	.93 .93

REFERENCES

1. Lunar Missions and Exploration, C.T. Leondes ed., Wiley and Sons: New York, New York, 1964.
2. Apollo Mission Reports.